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COMMENT

On the hull of two-dimensional percolation clusters

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Abstract. A recently proven equivalence between certain random walks with memory and the hull of percolation clusters in two dimensions is used to estimate the fractal dimension of the latter by means of Monte Carlo simulations. We find $D_H = 1.750 \pm 0.002$, in agreement with a recent conjecture that $D_H = \frac{7}{4}$ exactly.

In the ongoing programme to determine critical exponents related to percolation, considerable effort has recently been devoted to the exterior perimeter or 'hull' of 2D percolation clusters (Voss 1984, Ziff *et al* 1984, Weinrib and Trugman 1985, Kremer and Lyklema 1985a, b, Sapoval *et al* 1985, Rosso *et al* 1985, Bunde and Gouyet 1985, Gunn and Ortuño 1985). Although the 'main' critical exponents of percolation (those related to the exponents of the Potts model in a field theoretic formulation) are believed to be known exactly (Stauffer (1985); in particular, one has $\nu = \frac{4}{3}$ and $\beta = \frac{5}{36}$), other exponents like the fractal dimension D_H of the hull are only known approximately.

Numerical values of D_H reported in the above papers are all very close to $D_H = 1.75$. It was thus conjectured in Sapoval *et al* (1985) and Bunde and Gouyet (1985) that $D_H = \frac{7}{4}$ exactly. This is, however, not in good agreement with the most precise estimate $D_H = 1.764 \pm 0.009$ of Kremer and Lyklema (1985b). It seems thus that a new precise determination of D_H would be welcome.

Another personal motivation for studying this problem was the observation of Ziff *et al* (1984) and Weinrib and Trugman (1985) that the hull of percolation clusters can be related to certain self-avoiding walks with memory (I have used this already when quoting the results of Kremer and Lyklema, since they indeed studied walks which should be in the same universality class as the above walks, but which are not obviously related to percolation). Although this relation between walks and percolation seemed to be closely related to the duality properties of percolation, it was strange that the above authors had not worked it out for bond percolation on the square lattice which is the simplest lattice with the simplest duality properties. Instead, Ziff *et al* had dealt only with site percolation on the square lattice, while Weinrib and Trugman had discussed only site percolation on the triangular lattice. Only after having done most of the simulations did I realise that this relation had already been established independently by Gunn and Ortuño (1985).

According to the latter reference, the hulls of bond percolation clusters on a square lattice are constructed in the following way. Start with a walk on a randomly chosen site and move one step along one of the axes. Then turn either left or right with equal probability, and remember the sense of turning at that site. Continue that way until the walk either escapes towards infinity or until it closes by using the first (starting)

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bond. If the walker arrives during the walk at a previously visited site, take the same turn (left/right) as taken the previous time. Notice that the walk has to turn at every step; straight continuations are not allowed. It can be shown that this walk is self-avoiding in the sense that no bond is crossed twice, while sites can never be visited more than two times.

The simulation of such walks is fast (14 μs/step on a CDC 170/175). Furthermore, on an infinite lattice the chance to 'survive' n steps before closing a loop decreases only very slowly with n, like n^{-c} with $c=(\beta/\nu)D_H=0.06$. Thus it is easy to generate rather long walks.

For the present simulations, we took square lattices of size $L\times L$ with periodic boundary conditions. Thus walks can either close by making a full left or right turn, or they can wrap around the torus in which case the net turn is zero on lattices with even L. We concentrate on the latter walks. They have a characteristic length of order L, and we thus expect the average number of steps before closing to increase as

$$\langle n\rangle \sim L^{D_H}. \tag{1}$$

In order to test (1), we used lattices with L up to 360. The average number of steps for that L was $\sim 2\times 10^4$, so that we can hope that we do not have any corrections to scaling to struggle with. This hope is vindicated by the results shown in figure 1. They were obtained with typically $\sim 2\times 10^5$ to $\sim 4\times 10^5$ independent trials per L (except for very large L: 120 000 trials for $L=120$, 80 000 for $L=180$ and $L=300$, 40 000 for $L=360$). In order to compare our results with the conjecture $D_H=\frac{7}{4}$, we show there the quantity $\langle n\rangle/L^{1.75}$. We see that this quantity is indeed constant for $L>10$, giving the estimate

$$D_H=1.750\pm 0.002. \tag{2}$$

This is in perfect agreement with the conjecture, and thus disagrees somewhat with the less precise estimate of Kremer and Lyklema (1985b).

The results $D_H=\frac{7}{4}$ was indeed derived heuristically by Bunde and Gouyet (1985). Our numerical result suggest that their arguments were indeed basically correct.

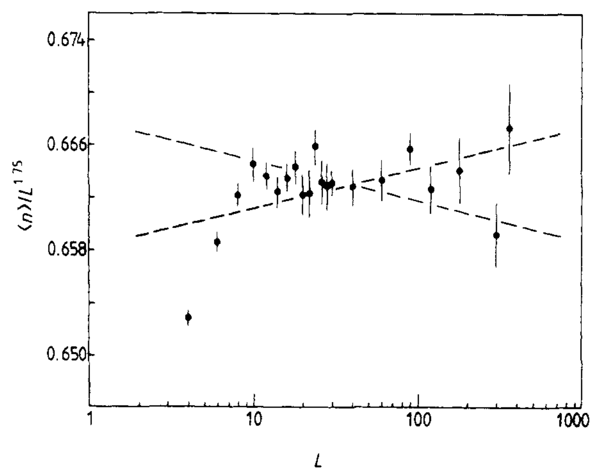

Figure 1. The average number of steps on periodic $L\times L$ lattices needed to close walks with zero net winding number. In order to eliminate the dominant behaviour, the data were divided by $L^{1.75}$. The angle enclosed by the broken lines corresponds to the quoted uncertainty of D_H.

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